GHOST HUNT · A Quantum Search Game

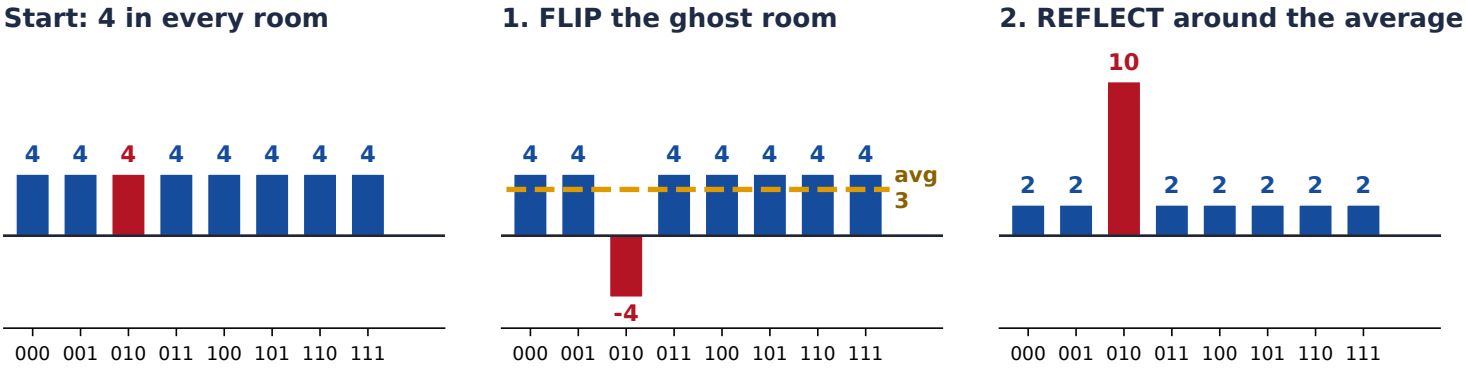
2 players or 2 teams · one class period · grades 6–8 · runs Grover's real quantum search algorithm

A ghost hides in one room of a hotel. A normal computer must open rooms one at a time. A quantum computer sends a search wave into every room at once. It cannot look at the wave, but each round of "flip and reflect" piles the wave up in the ghost's room. Then you LOOK, and the wave lands in one room.

The hotel

8 rooms, numbered in binary: 000, 001, 010, 011, 100, 101, 110, 111 (3 qubits make 8 rooms). Every room starts with a wave of 4.

One round = FLIP, then REFLECT



1. FLIP: the ghost room's wave changes sign. (Only the hider knows which room. You don't need to know!)
2. AVERAGE: add all 8 rooms and divide by 8. Here $(7 \times 4 - 4) \div 8 = 3$.
3. REFLECT: every room's new wave = 2 × average − old wave. The flipped room shoots up; the others shrink.
4. CHECK: the squares of all 8 waves always add up to 128. Here $10^2 + 7 \times 2^2 = 128$. If not, find your mistake.

Shortcut: all empty rooms stay equal, so you only track two numbers: G (each ghost room) and E (each empty room).

LOOK (measure)

Flip 7 coins in a row. Heads = 1, tails = 0. Read them as a binary number (place values 64, 32, 16, 8, 4, 2, 1), from 0 to 127. If the number is LESS than (number of ghosts) × G^2, you caught a ghost. Otherwise you opened an empty room. Either way, the wave is gone.

GHOST DUEL

1. The hider secretly writes 1, 2 or 3 ghost rooms on a slip.
2. The hunter secretly writes how many rounds to run: 0, 1, 2 or 3.
3. Reveal: the hider says HOW MANY ghosts (not where). The hunter shows the rounds.
4. Together, fill in the Wave Tracker for that many ghosts and rounds. Check 128 after every round.
5. The hunter flips 7 coins and LOOKS. A catch scores 1 point. Swap roles. Play 6 nights; most points wins.

Watch out: one more round is not always better. Too many rounds push the wave back out of the ghost room!

Warm-up: with 4 rooms (00, 01, 10, 11), start at 2 each. One round of flip and reflect gives the ghost room 4 and the others 0, so you always catch it. Grover's search (1996) is one of the most famous quantum algorithms. A million rooms need about 800 rounds; a normal search needs about 500,000 looks.

GHOST HUNT · Wave Tracker

Names: _____ vs _____

With M ghosts in 8 rooms: average = (M × (−G) + (8 − M) × E) ÷ 8

new G = 2 × average + G new E = 2 × average − E

Check after every round: $M \times G^2 + (8 - M) \times E^2 = 128$

Night 1 · hunter: Player 1 · ghosts M = ____ · rounds = ____

round	G (ghost)	E (empty)	average	check = 128?
start	4	4	—	128
1				
2				
3				

64	32	16	8	4	2	1

coins → number: _____
ghost if less than $M \times G^2$

caught? yes / no

Night 2 · hunter: Player 2 · ghosts M = ____ · rounds = ____

round	G (ghost)	E (empty)	average	check = 128?
start	4	4	—	128
1				
2				
3				

64	32	16	8	4	2	1

coins → number: _____
ghost if less than $M \times G^2$

caught? yes / no

Night 3 · hunter: Player 1 · ghosts M = ____ · rounds = ____

round	G (ghost)	E (empty)	average	check = 128?
start	4	4	—	128
1				
2				
3				

64	32	16	8	4	2	1

coins → number: _____
ghost if less than $M \times G^2$

caught? yes / no

Night 4 · hunter: Player 2 · ghosts M = ____ · rounds = ____

round	G (ghost)	E (empty)	average	check = 128?
start	4	4	—	128
1				
2				
3				

64	32	16	8	4	2	1

coins → number: _____
ghost if less than $M \times G^2$

caught? yes / no

Night 5 · hunter: Player 1 · ghosts M = ____ · rounds = ____

round	G (ghost)	E (empty)	average	check = 128?
start	4	4	—	128
1				
2				
3				

64	32	16	8	4	2	1

coins → number: _____
ghost if less than $M \times G^2$

caught? yes / no

Night 6 · hunter: Player 2 · ghosts M = ____ · rounds = ____

round	G (ghost)	E (empty)	average	check = 128?
start	4	4	—	128
1				
2				
3				

64	32	16	8	4	2	1

coins → number: _____
ghost if less than $M \times G^2$

caught? yes / no

Score: Player 1 _____ Player 2 _____

GHOST HUNT · Explore the Sweet Spot

1. Fill in the chance of catching a ghost ($M \times G^2 \div 128$) for each number of ghosts and rounds in an 8-room hotel. Use your Wave Tracker. Write it as a fraction of 128 or a percent.

	0 rounds	1 round	2 rounds	3 rounds
1 ghost				
2 ghosts				
3 ghosts				

2. Which number of rounds is best for 1 ghost? What happens if you do one round too many?

_____

3. In the duel you don't know how many ghosts there are. Which choice is the safest? Which is a gamble?

_____

4. Warm-up hotel with 4 rooms (00, 01, 10, 11): start at 2 in every room, so the squares add to 16. Do one round with 1 ghost:

round	G (ghost)	E (empty)	average	check = 16?
start	2	2	—	16
1				

5. The million-room hotel (use the computer: the Ghost Hunt browser lab or a spreadsheet). Find the best number of rounds for each hotel and compare with a normal search, which needs about half the rooms on average.

rooms	best rounds	catch chance	normal search looks
4			2.5
16			8.5
64			32.5
1,024			512.5
1,048,576			524,288.5

6. Rooms 4 → 16 → 64 → 1,024: what happens to the best number of rounds each time the rooms are multiplied by 4? Hint: try the square root of the number of rooms.

_____

GHOST HUNT · Teacher Key

ghosts	rounds	G	E	average	check	catch chance
1	0	4	4	—	128	16/128 = 12.5%
1	1	10	2	3	128	100/128 = 78.1%
1	2	11	-1	1/2	128	121/128 = 94.5%
1	3	13/2	-7/2	-9/4	128	42.25/128 = 33.0%
2	0	4	4	—	128	32/128 = 25.0%
2	1	8	0	2	128	128/128 = 100.0%
2	2	4	-4	-2	128	32/128 = 25.0%
2	3	-4	-4	-4	128	32/128 = 25.0%
3	0	4	4	—	128	48/128 = 37.5%
3	1	6	-2	1	128	108/128 = 84.4%
3	2	-1	-5	-7/2	128	3/128 = 2.3%
3	3	-13/2	-1/2	-11/4	128	126.75/128 = 99.0%

Best single look: 1 ghost → 2 rounds (94.5%); 2 ghosts → 1 round (100%); 3 ghosts → 1 round (84.4%) or 3 rounds (99.0%). With an unknown number of ghosts, best play is about 94% '1 round' and 6% '2 rounds' for the hunter, and about 83% '1 ghost' and 17% '3 ghosts' for the hider; the hunter then catches a ghost 79% of the time. A look after 0 rounds is a normal (classical) guess: 1/8, 2/8 or 3/8.

rooms	best rounds	catch chance	√rooms	normal looks
4	1	100.0%	2.0	2.5
8	2	94.5%	2.8	4.5
16	3	96.1%	4.0	8.5
64	6	99.7%	8.0	32.5
1,024	25	99.9%	32.0	512.5
1,048,576	804	100.0%	1,024.0	524,288.5

Pattern: best rounds ≈ 0.785 × √rooms (π/4 × √N). Multiplying the rooms by 4 doubles the rounds, while a normal search needs 4 times as many looks. The 2-number shortcut is exact: every empty room always has the same wave, so the class can do the arithmetic in the open without learning which room is haunted. The paper rule matches the real Grover circuit (Hadamard gates, oracle, diffusion) for every hiding place in 4-, 8- and 16-room hotels. The 7-coin look matches the exact chance to within 0.6%.